Classification of Signals

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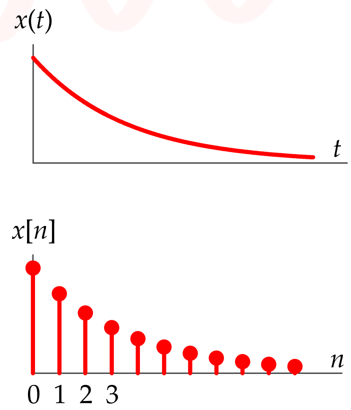


Continuous-time vs. discrete-time signals

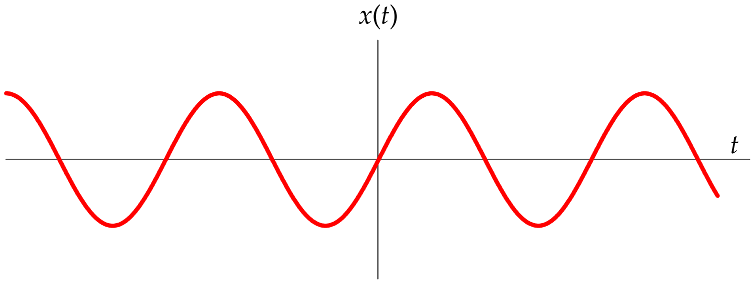
Continuous-time signal: $x(t)$ is specified for a continuous set of time instants t (i.e., t is a continuous variable).

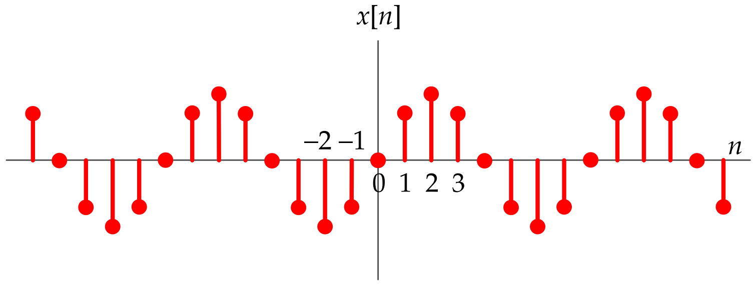
Discrete-time signal (or sampled signal): $x(t)$ is specified only at discrete time instants t (i.e., t is a discrete variable).

Discrete-time signals are often expressed as a **sequence** of numbers, denoted by $x[n]$ or $\{x_n\}$, instead of $x(t)$, where n is an integer.

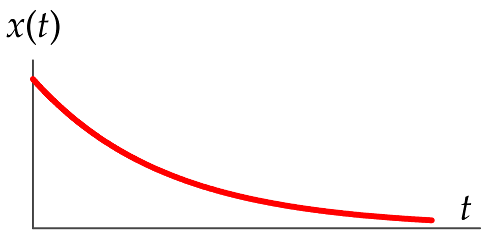
The concept of continuous-time vs. discrete-time signals should not be confused with the concept of analog vs. digital signals.

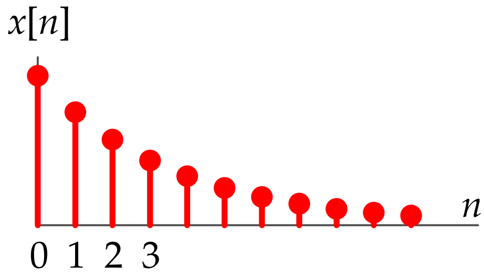
Continuous-time vs. discrete-time signals





Continuous-time vs. discrete-time signals





Discrete-time $x[n]$ may represent an inherently discrete phenomenon, e.g., number of students registered for this class every semester.

Alternatively, $x[n]$ may be obtained by sampling a continuous-time signal $x(t)$. These are called **samples**,

$$x(t_0), x(t_1), x(t_2), \dots, x(t_n), \dots = x[0], x[1], x[2], \dots, x[n], \dots$$

In other words,

$$x[n] = x_n = x(t_n)$$

If the **sampling interval** or **sampling time** T_s (i.e., the time between successive samples) is equal for all samples (uniform sampling), then,

$$x[n] = x_n = x(nT_s)$$

Analog vs. digital signals

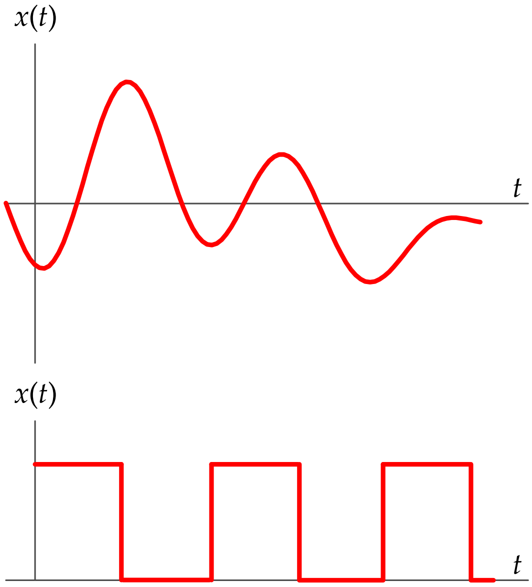
Analog signal: $x(t)$ is continuous-time and can assume any value within a continuous interval (a, b), which can be $(-\infty, \infty)$.

Digital signal: $x(t)$ is continuous-time but can assume only a finite number of distinct values (two levels in binary or more in M-ary).

Continuous-time vs. discrete-time refers to the time (horizontal) axis, while analog vs. digital refers to signal amplitude (vertical axis).

Notice that we did not mentioned discrete-time signals, because typically the discrete-time signal is the intermediate stage between converting the signal from analog to digital (when this conversion is performed).

Analog vs. digital signals



Real vs. complex signals

Real signal: All values of $x(t)$ are real numbers.

Complex signal: Values of $x(t)$ can be complex numbers. Remember that a complex number can be a purely real number, purely imaginary number or a combination of real and imaginary values ($\mathbf{Z} = a + jb$).

Hence, strictly speaking, all real signals are special cases of complex signals, where the complex number is a purely real number.

We make the distinction, however, because real-life physical quantities, such as voltage and current, are always real signals, and never complex signals. Complex signals are useful mathematical tools, such as $e^{j\omega t} = \cos(\omega t) + j \sin(\omega t)$, simplifying Fourier series.

Deterministic vs. random signals

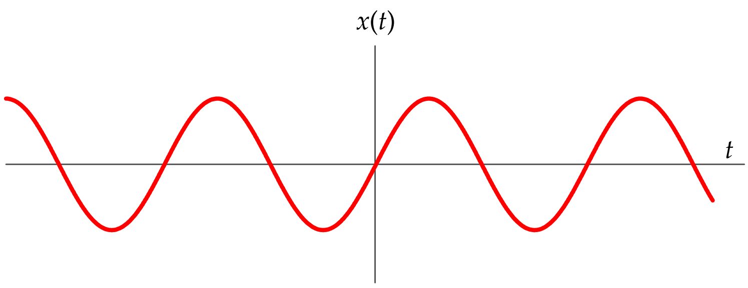
Deterministic signal: $x(t)$ description is known completely, in either mathematical form or graphical form. For example the following $x(t)$ are deterministic signals

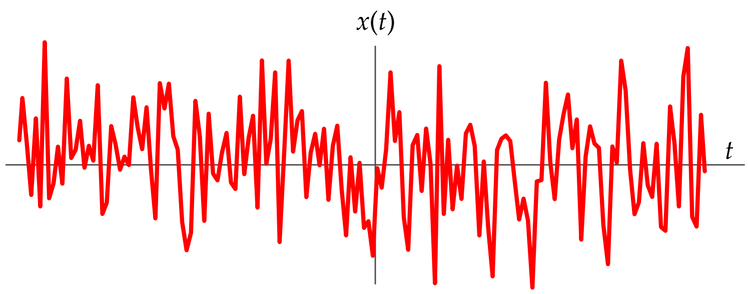
$$x(t) = A\cos(\omega t)$$

$$x(t) = A\,\mathrm{rect}\left(\frac{t}{\tau}\right)$$

Random signal: Values of $x(t)$ at any given time are random values, similar to flipping a coin. The random signal $x(t)$ cannot be modeled using an equation, rather it is characterized probabilistically. The most popular random signal is noise $n(t)$.

Deterministic vs. random signals





Periodic vs. aperiodic signals

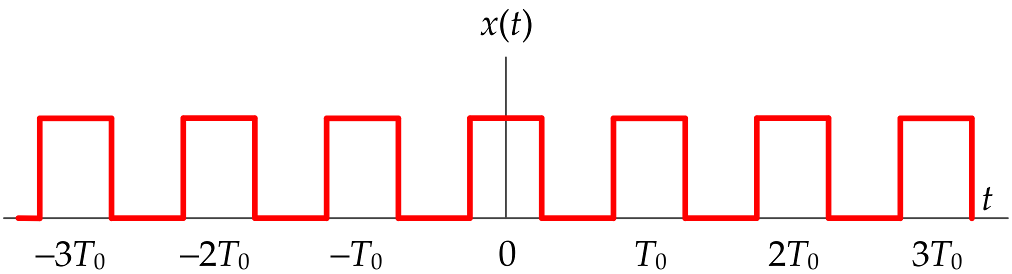
Periodic signal: $x(t)$ is said to be periodic with period T if there is a positive nonzero value of T, for which,

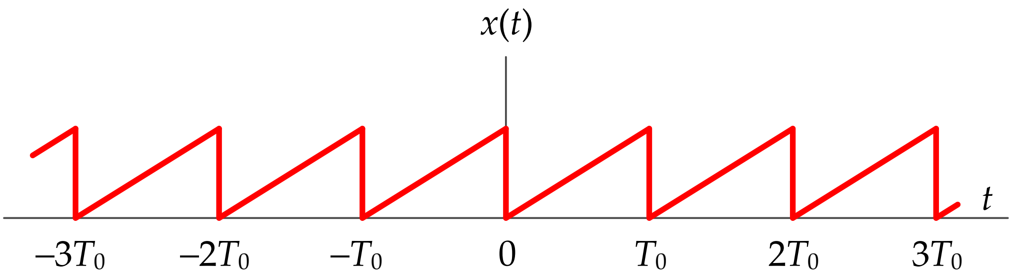
$$x(t+T) = x(t) \qquad \forall t$$

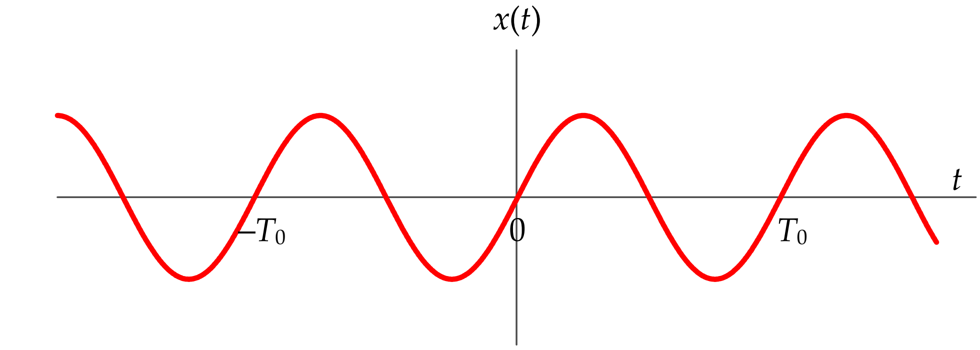
In other words, the signal keeps repeating itself every period T (seconds). The smallest positive value that satisfies the above equation is called the **fundamental period** T_0. Typically, we use the word period to mean the fundamental period.

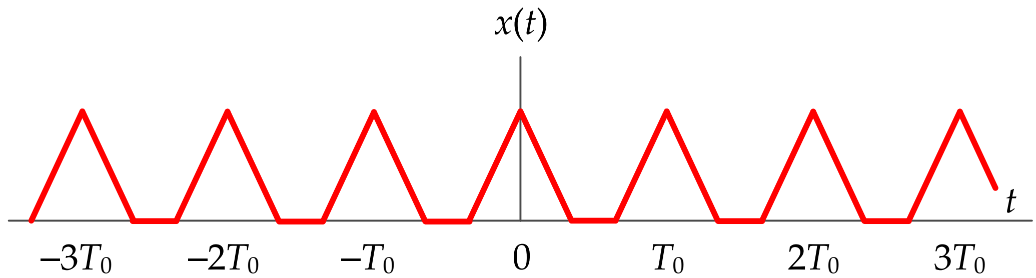
Aperiodic signal: $x(t)$ does not satisfy the above condition. It is called non-periodic or aperiodic signal.

Periodic signals

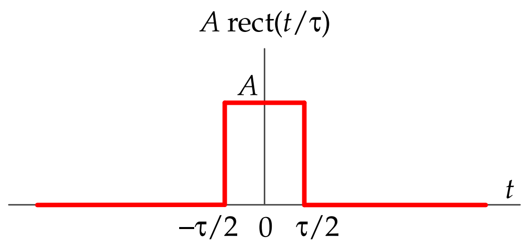


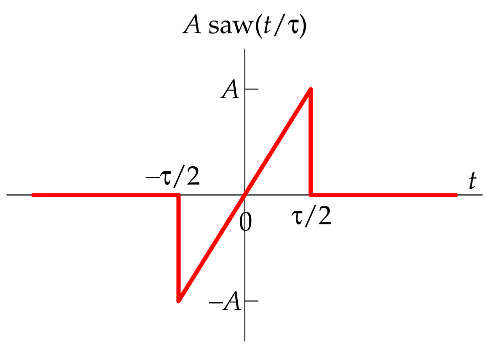


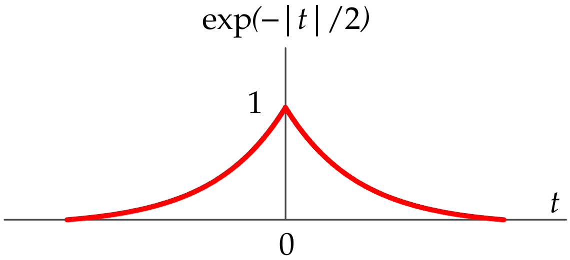


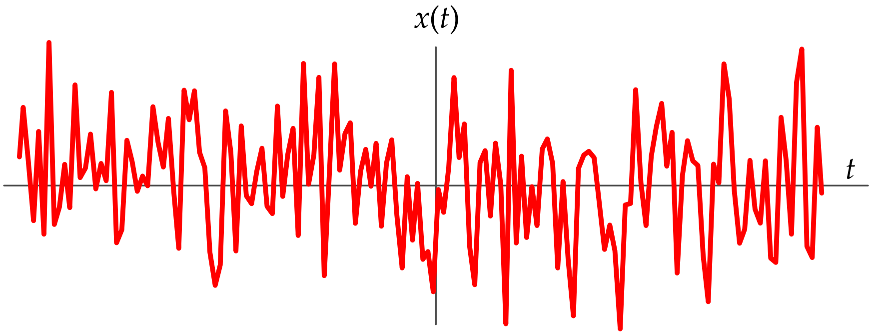


Aperiodic signals



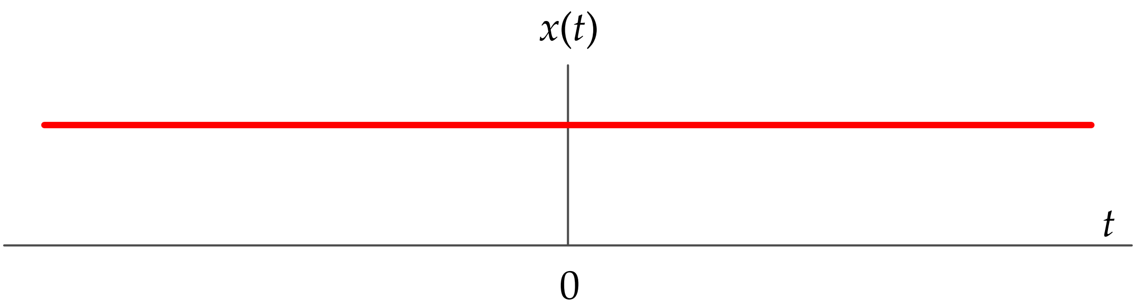






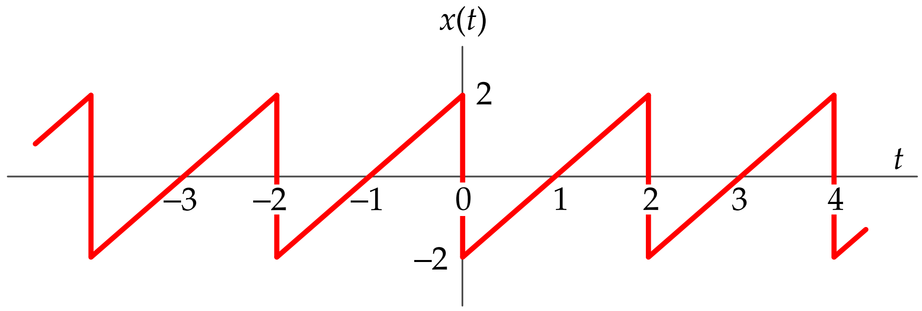
The DC signal (constant value) is a special case, since the condition $x(t+T) = x(t)$ applies for any choice of period T, (and so there is no smallest positive value T_0).

We do not really care if we call the DC signal periodic or aperiodic. It is just a special case.



A useful property of periodic signal $x(t)$ with fundamental period T_0 is that the area under $x(t)$ over any interval of duration T_0 is the same

$$\int_0^{T_0} x(t)dt = \int_{-T_0/2}^{T_0/2} x(t)dt = \int_{-T_0/4}^{3T_0/4} x(t)dt = \int_{t_0}^{t_0+T_0} x(t)dt = \int_{T_0} x(t)dt$$



The frequency of the periodic signal is:

$$f = \frac{1}{T_0} \quad [Hz]$$

$$\omega = 2\pi f = \frac{2\pi}{T_0} \quad [rad/s]$$

From now on we call this frequency the **fundamental frequency**:

$$f_0 = \frac{1}{T_0} \quad [Hz]$$

$$\omega_0 = 2\pi f_0 = \frac{2\pi}{T_0} \quad [rad/s]$$

This fundamental frequency f_0 turns out to be the frequency of the first harmonic $f_1 = f_0$ (first component in the Fourier series).

But the periodic signal also contains other harmonics with frequencies:

$$f_2 = 2f_0 \qquad \text{(second harmonic)}$$

$$f_3 = 3f_0 \qquad \text{(third harmonic)}$$

$$\cdots$$

$$f_n = nf_0 \qquad (n^{\text{th}} \text{ harmonic})$$

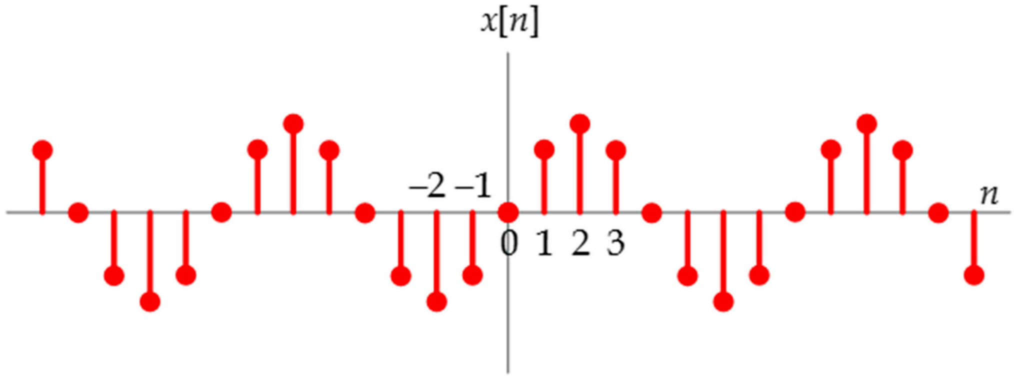
$$\cdots$$

So, we need to make the distinction. Hence, the fundamental frequency f_0 is the frequency of the periodic signal and also the first harmonic.

Periodic discrete-time signals $x[n]$ satisfy for positive integer N

$$x[n+N] = x[n] \qquad \forall n$$

The fundamental period N_0 of $x[n]$ is the smallest positive integer N for which the above equation holds.



A sequence obtained by uniform sampling of a periodic continuous-time signal might not be periodic. Think about a condition where this is the case. **Hint:** Think about the relationship between T_0 and T_s.

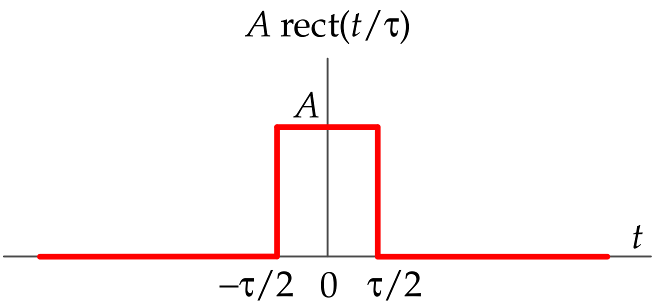
The sum of two continuous-time periodic signals may not always be periodic. Why?

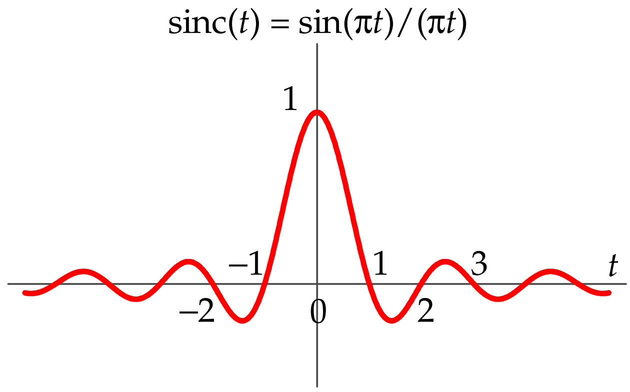
The sum of two periodic sequences is always periodic.

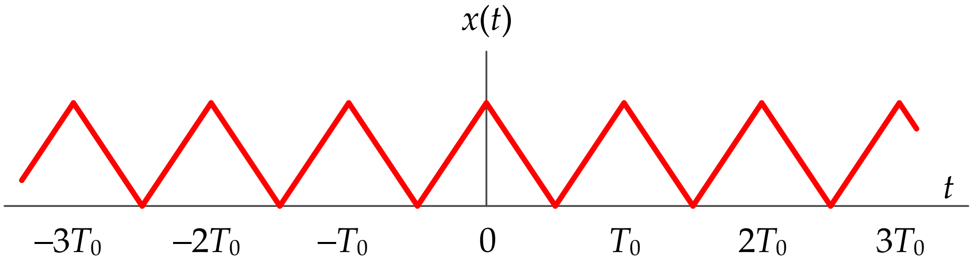
Even vs. odd signals

Even symmetry: $x(t)$ is even signal, if it is symmetric with respect to the vertical axis, meaning that its graph remains unchanged after reflection (via a mirror) about the vertical axis. Hence, even signal $x(t)$ or $x[n]$ satisfies the condition

$$x(-t) = x(t)$$
$$x[-n] = x[n]$$

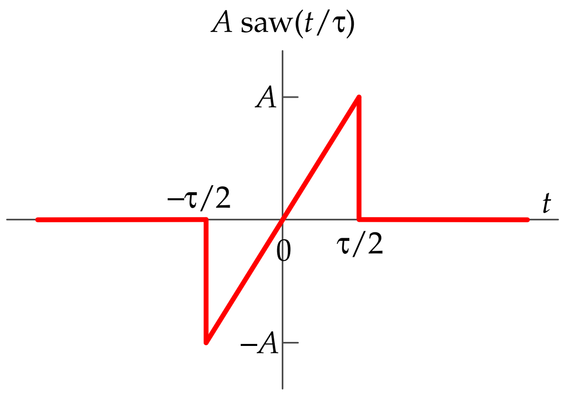




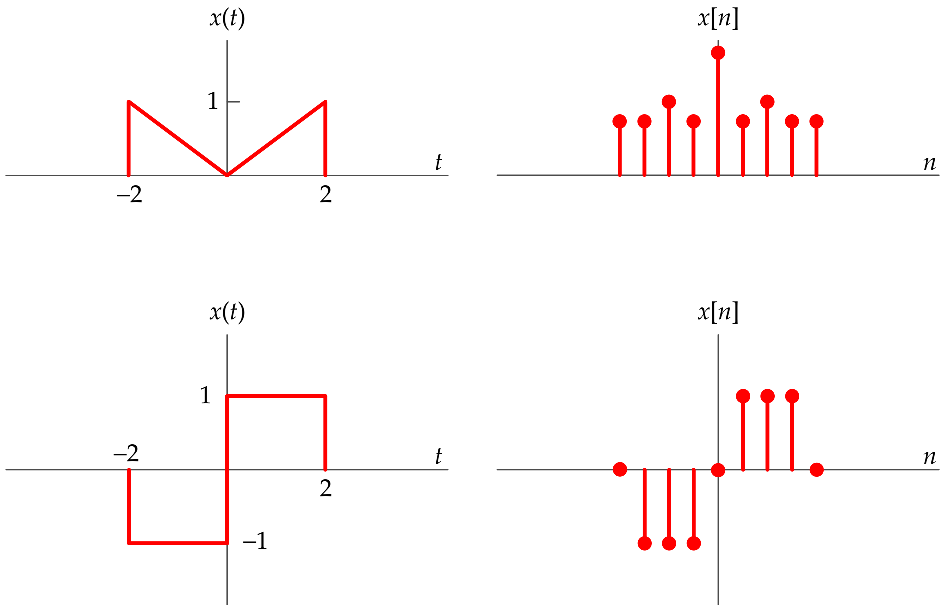


Odd symmetry: $x(t)$ is odd signal, if it has rotational symmetry with respect to the origin, meaning that its graph remains unchanged after rotation of 180 degrees about the origin, which is equivalent to reflection (via a mirror) about the vertical axis followed by another reflection (via a mirror) about the horizontal axis. Hence, odd signal $x(t)$ or $x[n]$ satisfies the condition

$$x(-t) = -x(t)$$
$$x[-n] = -x[n]$$



What about these signals?



Exercises

Q1. Is $A\cos(\omega t)$ even signal or odd signal or neither?

Q2. Is $A\sin(\omega t)$ even signal or odd signal or neither?

Q3. Is $A\cos\left(\omega t - \frac{\pi}{8}\right)$ even signal or odd signal or neither?

Q4. Come up with two more examples of even signals and two more examples of odd signals?

Knowing that the signal is even or odd can simplify the calculations for Fourier series (see later).

Energy vs. power signals

From electric circuits we know that the **instantaneous power** absorbed by a resistor is related to the voltage across the resistor $v(t)$ and the current that passes through the resistor $i(t)$ as follows

$$p(t) = v(t) \times i(t) = \frac{v^2(t)}{R} = R \times i^2(t)$$

From now on, we will assume a normalized resistor $R = 1\ \Omega$. We typically solve using this system, then introduce R later. Hence,

$$p(t) = v^2(t) = i^2(t)$$

Or generally, the **instantaneous power** [Watt] in a signal $x(t)$ is

$$p(t) = x^2(t)$$

From instantaneous power, we can calculate the **average power** [Watt] in the signal $x(t)$ as

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} p(t)\, dt = \overline{p(t)}$$

$$P_x = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} x^2(t) dt = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |x(t)|^2 dt = \overline{|x(t)|^2}$$

This is similar but slightly different than the **total energy** [Joule] in the signal $x(t)$, which is calculated as

$$E_x = \int_{-\infty}^{\infty} p(t) dt = \int_{-\infty}^{\infty} x^2(t) dt = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

The absolute value in $|x(t)|^2$ is useful when $x(t)$ is a complex function, but does not affect calculations when $x(t)$ is a real function.

For a discrete-time signal $x[n]$, the **average power** [Watt] is

$$P_x = \lim_{N\to\infty} \frac{1}{2N+1} \sum_{n=-N}^{N} |x[n]|^2$$

and the **total energy** [Joule] is

$$E_x = \sum_{n=-\infty}^{\infty} |x[n]|^2$$

Energy signal: $x(t)$ or $x[n]$ is an energy signal (or sequence) if and only if it has finite energy. In other words, $0 < E_x < \infty$. This immediately means that $P_x = 0$ because of the division by infinite time interval.

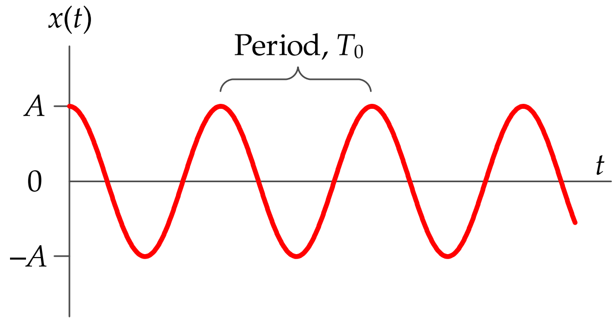
Power signal: $x(t)$ or $x[n]$ is a power signal (or sequence) if and only if it has finite and nonzero power. In other words, $0 < P_x < \infty$. This immediately means that $E_x = \infty$ because of the division by infinite time.

If neither property is satisfied, then the signal (or sequence) is neither energy signal nor power signal. The ramp signal is an example.

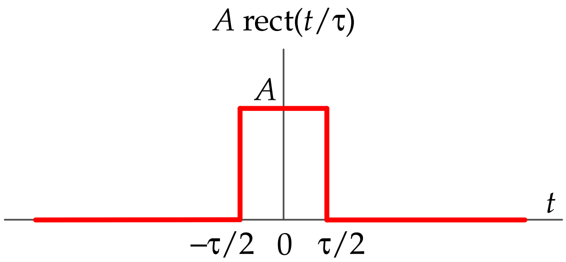
Hint: For the energy to be finite the signal amplitude must approach 0 as $|t| \to \infty$. When the amplitude does not approach 0 as $|t| \to \infty$, the signal energy is infinite.

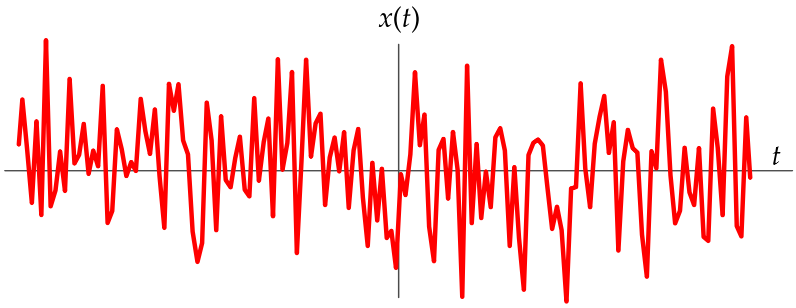
The **periodic** signal is a power signal if its energy content per period is finite. In that case, the average power need only be calculated over one period, without needing the limit,

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt = \frac{1}{T_0}\int_{-T_0/2}^{T_0/2} |x(t)|^2 dt = \frac{1}{T_0}\int_{0}^{T_0} |x(t)|^2 dt$$



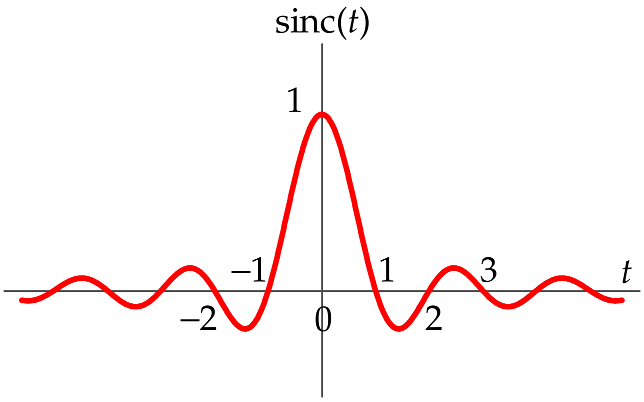
What about these signals?



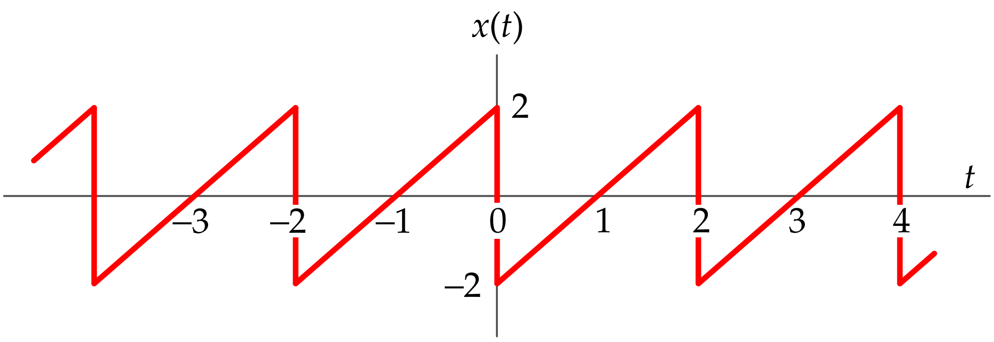


Q1. What is the average power in $x(t) = A\cos(\omega t + \varphi)$?

Q2. Is $x(t) = \text{sinc}(t)$ a power signal or energy signal? What is its energy E_x?



Q3. Calculate the average power P_x in the following signal $x(t)$. Is this signal a power signal or energy signal?



Answer:

$P_x = 4/3$ Watt. Power signal.

For the power signal, the signal magnitude does not approach 0 as $|t|$ approaches ∞, so this is a power signal. And it is periodic, so we can calculate based on one period only (without the limit)

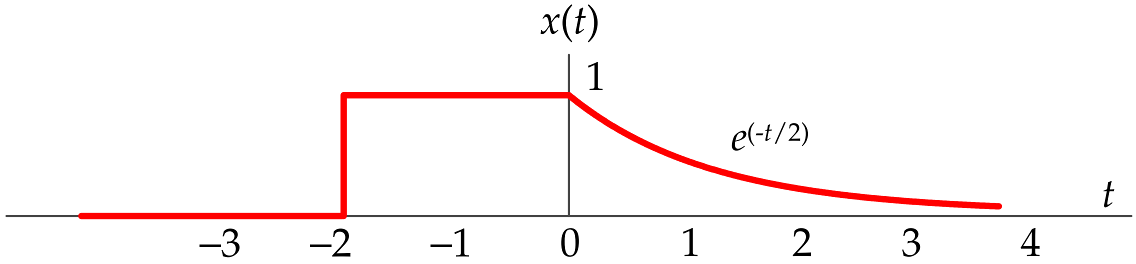
Let us consider he shifted signal first since power is unaffected by shift

$$P_x = \lim_{T\to\infty} \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt = \frac{1}{T}\int_{-T/2}^{T/2} |x(t)|^2 dt = \frac{1}{T}\int_{0}^{T} |x(t)|^2 dt$$

$$P_x = \frac{1}{2}\int_{-1}^{1} |x(t)|^2 dt = \frac{1}{2}\int_{-1}^{1} (2t)^2\, dt = \frac{4}{2}\int_{-1}^{1} t^2\, dt = \frac{4}{2}\left[\frac{t^3}{3}\right]_{-1}^{1}$$

$$= \frac{4}{2} \times \frac{(1)^3 - (-1)^3}{3} = \frac{4}{3}\ \text{[Watt]}$$

Q4. Calculate the energy E_x in the following signal $x(t)$. Is this signal a power signal or energy signal?



Answer:

$E_x = 2 + 1 = 3$ joule. Energy signal.

Q. Solution. For the energy signal, the signal magnitude approaches 0 as $|t|$ approaches ∞, so this is an energy signal

$$E_x = \int_{-\infty}^{\infty} |x(t)|^2 dt$$

$$E_x = \int_{-2}^{0} (1)^2 dt + \int_{0}^{\infty} \left(e^{-t/2}\right)^2 dt = [t]_{-2}^{0} + [-e^{-t}]_{0}^{\infty} = 2 + (-1) \times (0 - 1)$$
$$= 3\ [Joule]$$